Field perturbations in general relativity and infinite derivative gravity

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Chapter 1

Introduction

General relativity (GR) is a geometrical theory of gravity formulated by Albert Einstein in 1915 [1] and it is still considered the most accepted interpretation of gravity. In this theory it is recognised that space time can be represented using the Lorentz group, which encodes in it the idea of causality. This can only occur if there exists a maximum limit to how fast information can propagate, a crucial idea in GR which distinguishes it from Newtonian gravity. The action that describes the dynamics of the theory are given by the Einstein-Hilbert action written as

$$S = \int \sqrt{-g} \left[\frac{1}{2\kappa} R + \mathcal{L}_M \right] d^4x, \tag{1.1}$$

where $g = det(g_{\mu\nu})$, with $g_{\mu\nu}$ the metric that describes the space time. In this thesis we will work with metrics of a space like convention, that is they have a signature of $(-, +, +, +, ...)$. R is a Ricci scalar, $\mathcal{L}_m$ is the matter Lagrangian and $\kappa = 8\pi G c^{-4}$, with G the gravitational constant and c the speed of light.[1] Applying to this the principal of least action we obtain the Einstein field equations, given as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa \tau_{\mu\nu}, \tag{1.2}$$

where $G_{\mu\nu}$ is the Einstein tensor which has encoded in it the curvature of the space time, Λ is the cosmological constant and $\tau_{\mu\nu}$ is the stress-energy tensor. What this equation is stating is that the curvature experienced near an object is directly related to the matter content of that object, in fact the more matter the object has the greater the object will curve space and time [2]. One of the first experimental results which showed that GR is an attractive alternative to Newtonian gravity, is that GR was able to accurately explain the precession of the perihelion of the orbit of mercury, without the need for additional correcting factors, this is shown in Refs. [1, 3]. Another prediction of GR is that light should bend as it passes near massive objects. This is due to light always travelling along a geodesic, which is the shortest path between two points. In the case of Euclidean space this is simply a straight line connecting two points, in curved space this is not necessarily the case. As indicated in Eq. (1.2), the space around a massive object is curved, and as such the light will follow a curved path, as illustrated in Fig. 1.1. This bending of the light gives the impression that the object that is producing the light is located at a different point in space compared to where it actually is. In 1912 Einstein proposed that in the theory of general relativity the magnitude of this bending of light is twice that as predicted by von Soldner in 1804 [5, 6]. During the May 29 1919 solar eclipse Dyson et al. [7] observed the bending of light as predicted by general relativity. This observation solidified GRs position as a more acceptable interpretation of gravity. There are many

[1] In this thesis we will set $c = G = 1$ unless otherwise stated.

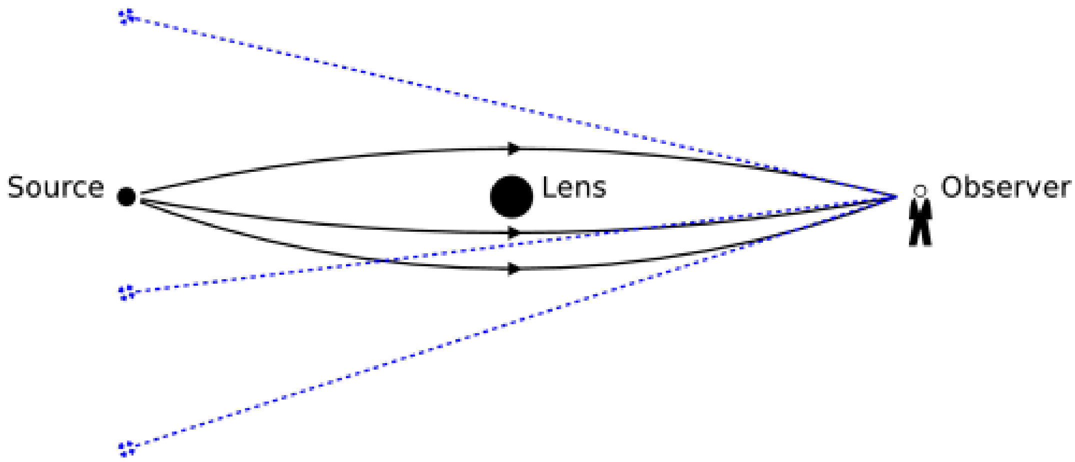

FIGURE 1.1: Image illustrating that light bends in the presence of a massive object, and that this bending of light can make it seems as if the source of light is at a different location Ref. [4].

more predictions and observations of the theory of GR, see Ref. [8] for a fuller list of observed phenomena in GR. These predictions and observations of GR are all in what is called the infrared (IR) limit of gravitational interactions, which is where the distance between interactions is large enough that quantum effects do not need to be considered.

There does, however, exist a problem with GR, and that is it cannot be incorporated into the standard theory of particle physics. As such it is not clear how the force mediator for gravity interacts with matter. This is contrary to how the other forces of nature are seen to interact. These theories come from quantum mechanical ideas, which are inherently probabilistic and do not necessarily consider large space time symmetries. They are predominantly interested in the short range interactions, that is at length scales where we need to consider the quantum nature of the interaction. These types of interactions occur at much higher energies as compared to the interaction occurring in the IR. As such, we say that these interactions occur in the Ultraviolet (UV) regime.

Due to the quantum mechanical nature of these theories, applying them to the matter part of the Einstein field equations suggests that there needs to be a quantisation of the space time part. This, however, leads to a non-renormalisable theory, meaning that there are infinities which we cannot control [9]. This non-renormalisability is indicative that the theory is only valid in the IR regime, and therefore cannot necessarily be trusted all the way up to the quantum level.

Another problem in GR is if it is applied to cosmology. In this context the theory predicts the necessary existence of singularities in any expanding universe that is spatially homogeneous and isotropic [10, 11]. This can be understood by considering an ever expanding universe with a constant matter content. If in the case of this universe we were to run the clocks backwards we would observe an ever increasingly dense universe, which would eventually become so dense as to be considered infinitely dense. Again, this would only occur in the small timescale interactions, or very early universe. This singularity is colloquially called the big bang singularity.

Our final motivation for considering theories beyond standard GR is that the Schwarzschild, Reissner-Nordström and Kerr metrics contain with in them at least one singularity. Notably there is a singularity in these metrics that cannot be removed through a coordinate transformation. As an example we consider the radial form of the Schwarzschild metric, which describes a non-rotating electrically neutral object [12],

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\theta^2 + r^2sin^2\theta d\phi^2, \tag{1.3}$$

where $f(r) = 1 - 2M/r$, with m the mass. Note that in the above we have used natural units, $c = G = 1$. We see that the two singular solutions are at $f(r) = 0$ and at $r = 0$. The singularity at $f(r) = 0$ occurs at $r = 2M$ and is located at the Schwarzschild radius, its existence suggests that there are two distinct regions in the space time. The coordinate singularity that occurs at this point can be removed be a reparameterisations of coordinates, however a check of the Killing vectors shows that there still exists two distinct regions of space [13]. The singularity located at $r = 0$, cannot be removed by a change of coordinates and this singularity is predicted to exist in every black hole. The consequences of this singularity might be that the predictability of physics and many other tools of physics would not work here. Fortunately, in 1969, Roger Penrose proved, within standard GR, the cosmic censorship conjecture, which states that in GR this singularity is always hidden by the event horizon, and as such cannot be observed in the physical universe [14]. This resolves the issue of predictability from a physical point of view, but does not resolve the issue in the theoretical framework of GR, since they still remain unresolved. It should be noted that this conjecture only applies to the singularities found in black holes and does not include the cosmological singularity.

The UV completeness problems of GR have plagued scientists for many years, and as such there have been numerous approaches to solving this problem see Refs. [15–18] for some of the ideas proposed to solve the problems in GR.

One of the proposed solutions is supersymmetric gravity, usually called super gravity (SUGRA). These theories propose that there exists a supersymmetric partner to the the graviton, which in the standard model is a spin-2 gauge boson, called the gravitino, which has a 3/2 spin. A full review on the topic is given by van Nieuwenhuizen et al. in Ref. [19]. [2] Some of these SUGRA theories take inspiration from string theory and require more than four dimensions to ensure that the theory is renormalisable. Some like the 11 dimensional SUGRA are still challenging to incorporate into the standard model [19]. However the study of higher dimensions is still a topic of interest, even in non-SUGRA models of gravity, since it can give us insight into which features are unique to four dimensional gravity [22]. In this thesis we explore these higher dimensions with this motivation in mind, rather than motivating the need for the existence of higher dimensional models of gravity in order to resolve issues in the theory.

Finally, the types of modified gravity that we will explore in this thesis are those containing higher order derivatives in their field equations. These theories modify the Einstein-Hilbert action by including higher orders of the curvature scalar, and in the hopes of obtaining a non-singular theory of gravity.[3] One such theory is that of fourth order derivative gravity which, has shown that the theory can be renormalised, at the expense of introducing ghosts into the theory [23]. In Sec. 1.3 we motivate these types of theories and briefly introduce the theory of Infinite Derivative Gravity (IDG), which is ghost free.

Before looking at the modified theories of GR we will look at a prediction of GR that has only recently been detected, namely gravitational waves. These were predicted

[2]Note that to date the LHC has not detected any supersymmetric particles [20, 21]

[3]In obtaining the non-singular gravity it is hoped that the issue of renormalisablity can be resolved, although this has not yet been done.

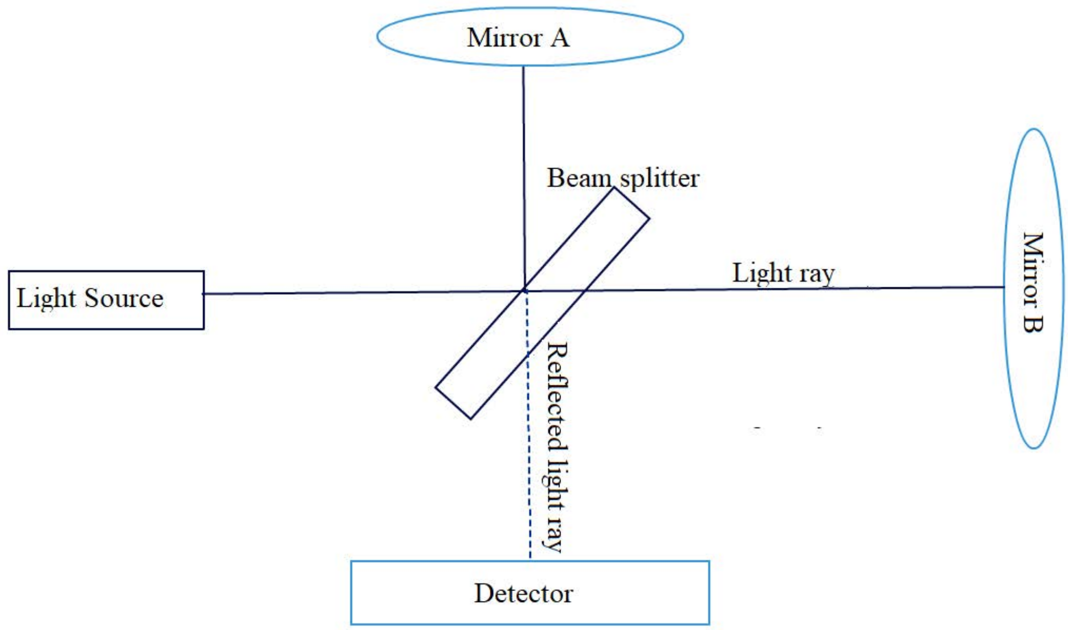

FIGURE 1.2: Diagram representing the setup of a Michealson-Morley inferometer experiment.

shortly after Einstein proposed his theory of GR, but have taken nearly a century to be observed directly [24].

1.1 Gravitational waves

Eq. (1.2) describes how the space time around an object curves due to the local energy and momentum [13]. Most importantly it ensures that the the conservation of momentum and energy are observed in GR [25]. If the object is accelerating, then these curvatures of the space time can propagate away from the object. Analogous to how an object moving through water creates a wake behind it. These propagating curvatures are called gravitational waves, and move away at the speed of light. Furthermore, the amplitude of these waves is a measure of how much they have stretched or contracted the space time as compared to the unperturbed space time. As they propagate away from the object they do carry with them some gravitational energy [26]. In fact their existence was inferred by observing the orbital decay of a binary pulsar system in 1982, since as the two objects orbited one another they slowly lost energy and thereby reduced the distance between each other, in the process increasing their orbital velocities [27].
Even though gravitational waves had long been predicted by GR they have only recently been discovered by the Laser Interferometer Gravitational-Wave Observatory (LIGO) research collaboration [28]. The first detection was on the 15th September 2015, with at least eight more detections since then. The experiment is very similar to the Michelson-Morley experiment which was used to prove that there exists no "aether" through which photons propagate, even though it was setup to try and detect this "aether" [29]. The Michealson-Morley experiment is setup as shown in Fig. 1.2, where a light source, in the case of LIGO a powerful laser, fires a light beam at a beam splitter, which sends two beams of light in perpendicular directions to each other towards two mirrors, in the case of LIGO these mirrors are 4km away from

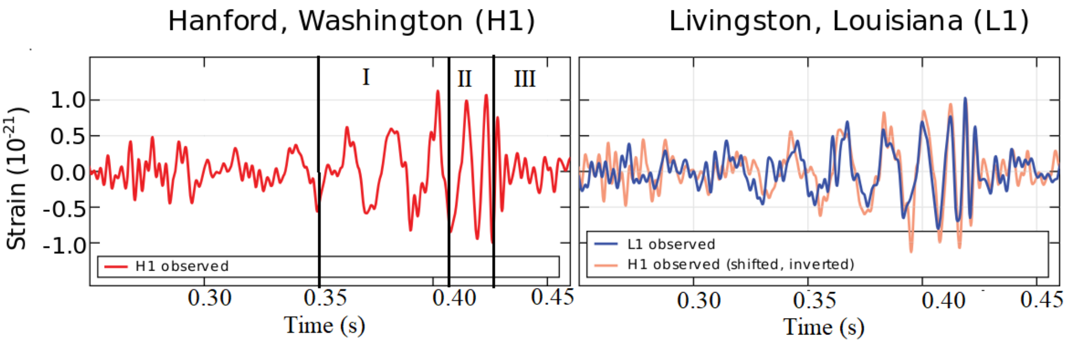

FIGURE 1.3: Gravitational wave event GW150914 detected by the LIGO Hanford, left column, and the LIGO Livingston, right column, detectors. [24]

the beam splitter. The light reflects off the mirrors and returns to the beam splitter, where the two laser beams are recombined and sent towards the detector. In recombining the two light beams interfere with each other, where in the case of LIGO the mirrors are placed such that the light beams interfere destructively, so that no photons are detected when the system is not detecting any gravitational waves. If a gravitational wave were to move through the LIGO detector, then it will cause the arms of the detector to either lengthen or contract by very small amounts. Since the arms are no longer the same length, the two light beams return to the beam splitter slightly out of phase. As such, they will no longer interact in such a way as to be completely destructively interfering with each other, i.e. the detector measures some non-zero intensity of light. The intensity of the light is directly related to the amplitude of the gravitational wave that passed through the detector.

Due to the weak nature of gravity, and a $1/r$ fall off in strength due to propagation, the gravitational waves detected had an extremely small amplitude, making them very difficult to detect. In fact, the detection by the LIGO team on the 15th of September was the detection of two black holes orbiting each other and then colliding, where the mass of one of the black holes had a mass of $35^{+5}_{-3}M_{\odot}$ and the other had a mass of $30^{+3}_{-4}M_{\odot}$ [30]. The event resulted in the lengths of the arms in the LIGO detector to change by less than the diameter of a proton [30]. Fig. 1.3 shows the gravitational wave that the LIGO team detected. Looking at the left most image in Fig. 1.3 we have labelled three regions of interest. Region I is where the two black holes are orbiting each other and are spiralling towards each other, this is called the pre-merger stage. The frequency of the gravitational wave is proportional to the orbital velocity of the two black holes. So moving from region I to II we see that the frequency increases since the two black holes are falling in towards each other and therefore increasing their orbital velocity. In region II we see that the frequency has increased drastically and the amplitude of the gravitational waves has increased. This region of the graph shows the merger phase of the two black hole systems. Here the two black holes are colliding with each other resulting in a highly perturbed black hole. Due to this highly perturbed state the new black hole will be emitting large amounts of energy as it tries to return to an almost spherical space. Note that due to rotations the black hole will be slightly elongated at its equator. This phase of large energy emission and return to a more spherical space is called the ring down phase and occurs in the region III. This ring down phase has the same form as we would see from a Quasi Normal Mode (QNM). These QNMs are of interest as they can reveal a deeper insight into black holes and GR. In fact the first studies of QNMs were done in the 1950's by Regge and Wheeler in Ref. [31] on the QNMs on the surface of a black hole after it had been perturbed. Where they wanted to probe the stability of Schwarzschild black holes by studying the QNMs. In the 1970s Vishveshwara pointed out that these oscillations would be emitted into the surrounding space time and propagate away as gravitational waves [32], which was confirmed by the LIGO detector.

1.2 QNMs

In general terms a QNM is a normal mode with some dampening term. They occur when some object is perturbed in some way, and then emits the energy of the perturbation away from itself. For instance the tapping of a knife on a wine glass will create QNMs as at the surface of the glass resonates and emits sound waves. Extending on this thought, the allowed frequencies and the dampening terms are determined by the composition of the glass and the contents within the glass. Such that glasses of wine with different amounts of wine ring at different frequencies, and for differing lengths of time. This means that in theory if we were to measure the frequency of the ringing we could determine the amount of wine in the glass. There are a variety of ways of perturbing, or creating perturbed black holes. For instance the creation of a black hole from a collapsing star or the collision of two black holes would create these highly perturbed systems. But we need not use such massive objects to create the perturbations, which could also be created by a single field, as Regge and Wheeler suggested [31]. In all of these cases the black hole would emit QNMs as it radiated away energy to return to a none perturbed state, with the perturbation caused by a single field, possibly giving us an insight into that fields gravitational interaction. As in the case of the of the wine glass, there are certain parameters of the black hole that determine the allowed values of the emitted QNMs. Fortunately black holes are parameterised by only a few properties namely the mass of the black hole, its electric charge and the rotational speed of the black hole.
Research has been done on the allowed QNMs for spin-1/2, spin-1 and spin-2 fields [33, 34]. However there is a gap in the literature as spin-3/2 fields have not been studied to a large extent. In Ref. [35] the QNMs for spin-3/2 fields are investigated for Schwarzschild black holes. We note that in this paper we have used the results of a paper by R. Camporesi and A. Higuchi [36] where they have studied the eigenfunctions of a Dirac operator on an N dimensional sphere. Using their results we could easily expand our analysis to any arbitrary number of spatial dimensions. This means we were able to determine the effective potential for a D dimensional Schwarzschild black hole. Giving us an opportunity to not only test the limits of the numerical methods we used to obtain the QNMs, but also to check if there are any unique features in 4 dimensional gravity.
We chose to use two methods to determine the allowed QNMs, one being the well established Wentzel–Kramers–Brillouin (WKB) method, and the other being the improved Asymptotic Iterative Method (AIM). We chose these two methods since the WKB method is a well established method for determining QNMs, which has been extended to 6th order by R. Konoplya to increase the accuracy of the method [37–39]. However, obtaining this 6th order result is quite complicated, and can be computationally expensive. We therefore have used the AIM as an alternative method for calculating the numerical values of the QNMs. This method is easier to implement and it is hoped that it is computationally less expensive. It has been shown by H.

Cho et al. that this method can be used as an alternative for calculating the allowed QNMs of black holes [40–42]. In the case of the D dimensional black hole we noticed that for dimensions $D \geq 8$ our numerical methods began to breakdown. It would therefore be of interest to establish if these methods still work in the higher dimensional cases with electrically charged black holes and those that are in AdS spaces. As such the purpose of the first part of this thesis will be to determine the effective potentials and allowed QNMs in higher dimensional Reissner-Nordstöm and Anti-de Sitter (AdS) space times.

1.3 Higher order derivative theories of gravity

As stated in the opening paragraphs, of this thesis GR is a very successful theory in the IR range, but breaks down for short distances and small time scales, i.e. in the UV regime. As such, any new theory of gravity will need to only modify the UV interactions and return to the limit of GR in long range interactions. There are many theories that attempt to do exactly this, such as string theory, [43], loop quantum gravity [44, 45], causal set dynamics [46, 47], emergent gravity [48], and infinite derivative gravity [49–52].[4]

In this thesis, we will mainly focus on a class of theories known as IDG. The simplest modifications will lead to quadratic in curvature, but infinite derivative, corrections to the Ricci scalar, Ricci tensor and the Riemann tensor [50, 51]. [5] In 1962 R. Utiyama and B. De Witt showed that in order to ensure that gravity would be renormalisable we would need to consider higher order derivative terms in the action [59], and in 1977 K. Stelle showed that these higher order derivative theories are indeed renormalisable [23]. Most importantly, however, it was expected that the theories could be constructed in such away that they would introduce a new scale to the theory of gravity which would correct the gravitational behaviour in the UV, but return to GR in length scales larger than the newly introduced scale. This could then solve early universe and interior black hole singularities, while still leaving weak gravitational interactions unchanged [49]. As such the interest in these theories as possible solutions to the problems in GR has only increased, a review on the subject is given by Claus Kiefer in Refs. [44, 45].[6]

We shall briefly introduce some of the ideas in these theories, before stating why we look at the special case of IDG. One of the simplest ways of generalising the action of gravity is to consider actions of the form

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + \mathcal{L}_m. \tag{1.4}$$

The simplest choice of $f(R)$ is setting it equal to R, thereby obtaining the standard Einstein-Hilbert action of GR, given in Eq. (1.1). Using the principal of least action will then give us the standard Einstein field equations as written in Eq. (1.2). These field equations have at most second order derivatives in them [62–64]. K. Stelle

[4] Note that there has been an increase in the interest in IDG [53–55].

[5] Indeed the simplest generalisation will be to include only corrections which are invariant under Ricci scalar, known as $f(R)$ theories of gravity [56]. These are not necessarily new theories, in fact they were first experimented with in 1918, however this was done more as a curiosity in determining what effects would appear when looking at higher order derivative theories of gravity [57, 58].

[6] Further reviews on the topic are given in Refs. [60, 61]

improved on this action by considering actions of the form [23],

$$S = \int d^4x \sqrt{-g} \left(\alpha R + \beta R^2 + \gamma R^{\mu\nu} R_{\mu\nu} \right), \tag{1.5}$$

where he showed for appropriate values of α, β and γ the theory can be renormalised. However for these values of the coefficients the theory has a negative energy propagating degree of freedom. This results in vacuum instabilities in the Minkowski space time as well as prevents unitary in the quantum regime. By looking at the spin-2 component of the propagator in this theory we see the signature of a Weyl ghost [23, 51]. The presence of a this Weyl ghost violates the conditions for stability and unitarity.

In fact any theory that introduces any finite number of higher derivatives (more than 2) of scalars, vectors and tensors will always end up having kinetic operators with extra poles, which could be ghost like. [7] However in 1989 Kuz'min [66] and in 1997 Tomboulis [67] showed that by considering an infinite series of higher derivatives gauge theories and gravitational theories can be made renormalisable. Using these ideas Biswas, Mazumdar and Siegel suggested that the only way to ensure a ghost free non-singular theory of gravity was to consider an action which contained within it an infinite number of derivative terms [49]. Furthermore, in Ref. [49] the authors showed that such a theory of gravity could be applied to cosmology, and provided a viable solution to the cosmological singularity problem. Where they consider a cosmological bounce scenario instead of a Big Bang singularity. The authors have argued that this theory can be made asymptotically free in the UV regime without violating fundamental idea of physics such as unitary [50]. The Lagrangian that the authors have considered in this case is of the form

$$f = R + R_{\mu_1\nu_1\alpha_1\beta_1} \mathcal{O}^{\mu_1\nu_1\alpha_1\beta_1}_{\mu_2\nu_2\alpha_2\beta_2} R^{\mu_2\nu_2\alpha_2\beta_2}, \tag{1.6}$$

where $\mathcal{O}^{\mu_1\nu_1\alpha_1\beta_1}_{\mu_2\nu_2\alpha_2\beta_2}$ is a differential operator which contains within it covariant derivatives and $g_{\mu\nu}$ the metric of interest.

Outline

This thesis is split into two parts. Part one of this thesis will investigate the allowed QNMs for spin-3/2 fields near GR metrics describing electrically charged black holes and black holes in a universe with a none zero cosmological constant. Part 1 is set out as follows, in Ch. 2 we provide the background information necessary to construct the effective potential describing our spin-3/2 fields in the relevant black hole backgrounds. We also provide the tools necessary to calculate the allowed QNMs using the effective potentials. In Ch. 3 we obtain the effective potential for the spin-3/2 fields in a D dimensional space-time with an electrically charged black hole. Using this potential we calculate and present the numerical values for the QNMs for various dimensions and electrical charges of the black hole. Finally we present the absorption probabilities associated with the QNMs, before concluding and beginning our investigation of black holes in (A)dS in Ch. 4. Again we will investigate the effect of dimension and in this case curvature on the allowed QNMs.

In part 2 of the thesis we investigate the modified theory of gravity called IDG. In

[7] Note also that Ostrogradsky showed that any non-degenerate Lagrangian dependent on time derivatives higher than the second order corresponds to a linearly unstable Hamiltonian [65].

Ch. 5 we provide the necessary background material for understanding IDG, as well as show that it is in fact a ghost free theory of gravity by constructing the fields equations and the propagator using a modified Einstein-Hilbert action. Then in Ch. 6 we construct and test the metric for an electrically charged object in the theory of IDG. Finally we construct the rotational metric for IDG. In Ch. 7 we show that this metric is indeed singularity free and can be reduced to the GR rotational metric in the appropriate limit.

In Ch. 8 we present our final concluding statements remarking on both the works from part 1 and part 2.

Part I

Quasi normal modes in black hole backgrounds